NOTE: This essay is formatted so that it can be printed as a booklet for teachers to copy and share with colleagues. (Choose booklet printing so that the order of the pages is correct.)

Math/Science Matters: Resource Booklets on Research in Math and Science Learning

Booklet 1
Cognitive Issues that Affect
Math and Science Learning

"The Moon's Taking Off!": How Children's Intuitive Theories Influence Math and Science Learning

By Tina A. Grotzer, Project Zero,
Harvard Graduate School of Education

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Summary Points

What are naive theories?

?Children actively make sense of the world.
?Their notions tend to be based on what they can perceive.
?They construct intuitive understandings.
?These understandings may be at odds with science and math knowledge resulting in misconceptions or "naive theories."

How do naive theories create challenges for learning?

?Some misconceptions are shared, others are highly individual.
?Naive theories are resistant to change.
?Unaddressed misconceptions may be carried into adulthood.

Why are naive theories so problematic for math and science learning?

?Math and science concepts are often counter-intuitive.
?Historically, math and science were taught as a set of rules. Ritualistically applying algorithms masks misconceptions.
?Misconceptions have been identified in many areas of math and science.

What lessons does this research provide for educators?

?We must attend to how children construct knowledge and the difficulties this creates for learning math and science.
?We need to both reveal and address children's naive theories.
?We need to identify, not pass on our own misconceptions.
?We must help children see how the scientific theories lead to more powerful explanations than their naive constructions.

What are naive conceptions? How can a learner's naive conceptions be addressed? How does this affect learning in math and science?

A child tells you that the earth is round, but when asked for an explanation, describes the earth's surface as flat with a round ball of sky hovering over it.[1] Graduating college seniors are unable to explain what creates the seasons. They express the belief that it has to do with the shape of the earth's orbit and find themselves at a loss to explain why it is summer in the southern hemisphere while it is winter in the north.[2] What is happening here? Why are the students' understandings different from that which they presumably learned in school? For the past two decades, educational researchers have explored these questions, shedding light on what is going on and what should be done about it.

Research shows that children actively attempt to make sense of the world around them.[3] Children construct notions of how things work based upon their observations and their limited understandings. This may be especially obvious to parents when a young child attempts to explain a newly observed phenomenon. A four-year-old child upon observing the moon low in the horizon during the day time remarked to herself, "There's that moon again. I wonder what it's doing there. It must be taking off." The child created an explanation to make sense of the information she had amassed.

By school age, children have managed to make a great deal of sense out of their world. They have evolved a "robust and

serviceable set of theories: about mind, about matter, about life, about self."[4] While much of what children figure out about their world is accurate, some of it is not. Some of their theories are nonscientific. These theories have been referred to as "naive" or "intuitive" theories and as misconceptions.

How do naive theories create difficulties for school learning, particularly in math and science?

Children's theories often collide with the math and science of school learning. They may result in theories about scientific phenomena that are very different from what scientists believe -- misconceptions about the shape of the earth or why objects in motion eventually stop, for instance. Children's naive theories in mathematics can result in confusions about how to apply algorithms and about what they really mean.[5] Students may evolve a notion of mathematics as a set of rules to be applied[6] and end up applying formulas without understanding what they mean -- resulting in nonsensical solutions.

This may create a confusing scenario for teachers and parents. We often encourage children to use their common sense. We expect them to try to make sense of information in the world around them -- to reason about it and come up with new understandings. After all, actively attempting to make sense out of one's world is an important characteristic of a good learner. As discussed in other essays in this series, active sense-making is important to constructing knowledge that is connected and provides deep understanding.[7]

Research into this issue shows that it is not the "sense-making" behavior in itself that is problematic. Instead, the problem seems to generate from three distinct areas and how they interrelate: 1) the type of sense-making children engage in and the nature of the resulting naive theories; 2) the nature of science and math as scientists and mathematicians know it; and 3) the way that intuitive conceptions have often been dealt with in school. Each of these issues will be considered in turn.

What does children's sense-making look like? What is the nature of the resulting theories?

Researchers have identified some key *characteristics* of naive theories.[8]

Characteristic #1: *Naive theories can be personal and idiosyncratic.* The theories represent how children internalize their experience and construct understanding of it. Each child is the keeper of a unique set of experiences. As children learn and make sense of their world, they evolve their own personal lens for interpreting events, thus different aspects of the same event may seem important to different children.

Characteristic #2: *Naive theories may be customized for each event explained.* While scientists seek coherency in their explanations and hold this as a criterion for whether an explanation is plausible, the need for coherence may not be perceived by children.[9] Children often hold "customized" explanations for the events that they can observe, may be

accepting of more than one explanation for an event, and may be unconcerned if these explanations are contradictory.[10] This adds to the difficulty of knowing what they know.

Characteristic #3: *Naive theories can be very stable and resistant to change.* These theories represent a child's way of making sense and the child will attempt to fit new information into an existing set of understandings rather than dispose of a theory that he or she finds sensible. Children tend to compartmentalize their personal learning from the formal instruction they receive at school. They are often quite comfortable with the sense that they have made and have no reason to discard their own serviceable theory. Therefore, they often revert to their original, naive theories when asked to apply their school learning to new areas. In the example above about the shape of the earth, the child repeats what was learned as the "right answer" in school, yet continues to use his own model of a flat earth with a sphere of sky to make sense of cosmic events.

Despite the idiosyncratic nature of the theories youngsters come up with, children do share certain *tendencies* that define the nature of their sense-making. These tendencies help to create some parameters for the kinds of theories that children are likely to evolve.[11] What are some of these tendencies?

Tendency #1: *Children's thinking tends to be perceptually dominated* or based upon what they can observe.[12] They learn most easily from the information that they can readily gather through their senses -- auditory, visual, kinesthetic, and tactile. However, what appears to be so is not always what is so. This phenomenon is referred to as the appearance/reality distinction. For example, the earth appears to be a flat plane with the sky above it rather than an oblate spheroid. Children often interpret what appears to be so as though it is so. For example, they believe that sugar disappears when it dissolves instead of existing in a particle form too small to see.[13] Or if five stones are placed in a line and another set of five stones is placed near by but stretched out so that the line appears longer, young children may report that the second line has more.[14] While adults can also be influenced by appearance and context, they have a greater ability to reason logically[15] and a greater information base to draw from so they don't make the same errors that children do.

Tendency #2: *Children tend to consider absolute properties or qualities* before those that involve an interaction between the elements of a system. For example, when an object burns, a child may see it as a property of the substance that is burning while a scientist views it as an interaction between the burning substance and oxygen.[16] Understanding an interaction involves greater levels of abstraction so this tendency makes developmental sense.

Tendency #3: *Children are inclined to assume a linear, unidirectional relationship* between events and effects that they observe. Therefore, children, upon noticing that a lot of frogs have appeared in a pond and then shortly after that

many birds have arrived, might assume that the first event caused the second. They would be less likely to consider other possible relationships between the arrival of the frogs and the birds -- such as the possibility that both were caused by a snail population explosion. Children are also less inclined to explore extended effects. Therefore, they might consider the effects of air pollution on breathing but would be less likely to consider extended effects, such as the contribution of air pollution to acid rain and subsequently, to the deterioration of marble or bronze statues.

Common tendencies such as these can lead to certain common misconceptions. Thus, while some naive theories can be personal and unique, others are shared. Research into shared, common misconceptions promises to provide support to educators in attempting to reveal and address children's misunderstandings.

The majority of the research on particular misconceptions exists in the sciences, particularly physics, perhaps because for so many people it is the most counter-intuitive discipline. However, there is a rapidly growing body of research in other areas of science and math as well as early work in other disciplines such as the social sciences.[17]

Why are naive theories of particular concern for math and science learning?

While children do hold intuitive theories in other subject areas,[18] the theories are especially problematic for math and science learning. This is because math and science: 1) tend to be counter-intuitive; and[19] 2) lend to the rigid and ritualistic application of algorithms if conceptual understanding is not a teaching goal.[20]

Science and math asks us to construct mental models of things that cannot be directly perceived[21] and in fact may be counter-intuitive to what is perceived. This is at odds with children's perceptual biases for interpreting phenomenon. Indeed, exercising our "common sense" often leads to Aristotelian models of how the world works -- those that were believed prior to the mid-seventeen hundreds -- not those that present-day physicists use to explain the world. Research demonstrates that even adults, including advanced level students in physics, hold "common-sense" conceptions that are at odds with the thinking of present-day scientists and mathematicians.[22] For example, when a ball is thrown into the air, many adults intuitively believe that there is a force acting upon the ball in the direction of the motion.[23] According to Newtonian physics, once the ball leaves the hand of the person who threw it, the only force besides a bit of drag provided by air resistance is that of gravity. Even college physics students can get caught up in what they observe rather than the principles of movement and force that they have been taught.[24] The counter-intuitive notion of much of present-day science has been referred to as the "uncommon common sense of science."[25]

Compounding the counter-intuitive nature of science and math is the issue of how we talk about our worlds. Our everyday language reflects our perceptual and intuitive understandings

of the world rather than our scientific or mathematical understandings.[26] For example, we tell children to put their coat on because it will make them warm, instead of telling them that it will reduce their heat loss -- thus reinforcing misconceptions about how insulation works. Everyday language tends to reinforce confusion about particular terms and the notion that math and science speak a foreign language.

A heavy focus on the application of algorithms may compound the problem of persistent misconceptions in math and science learning.[27] It is widely held that mathematics consists of rule-governed knowledge mastered through memorization of formulas and facts.[28] If students can apply the correct algorithm in the correct instance, teachers may assume that they understand what they are doing. When questions are posed in slightly different ways, students may reveal that they are rigidly and ritualistically applying such algorithms without a deep understanding of the processes symbolized by the formula. For example, students may be able to demonstrate that 7 minus 3 is equal to 4. However, when asked a question such as, "If Sue has 7 apples and Tom has 3, how many more does Sue have?" students may trip up because the minus sign asks students to compare in this instance -- not to take away.[29] Students need a flexible and deep understanding of the process of subtraction to succeed with the problem.

What role has schooling played in relation to intuitive theories?

In the past, schooling attempted to teach new concepts to take the place of the child's naive theories. It was assumed that the school taught concepts would supplant the child's naive theories. We now understand that this underestimates the power of children's naive theories.[30] Learners often cling to "naive conceptions" despite formal instruction. Children are able to parrot back what they were expected to learn, yet then revert back to their own models in explaining related concepts.

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1. From an early age, children develop ideas related to *how the world works* and what different science words *mean*.

2. These ideas are usually *strongly* held.

3. These ideas may be *significantly different* from how scientists view the world.

4. These ideas are *sensible* and *coherent* from a child's point of view.

5. Traditional teaching often *does little* to influence or change these ideas.

Prior to this research, educators may have been unaware not only of the misconceptions held by their students, but of those that they themselves held. One can assume that those who taught our current teachers were equally unaware of such

misconceptions and how resistant they can be. Thus, it is not surprising that many adults hold naive conceptions similar to those held by school age children.[31] For example, a teacher may present the concept of heat as energy transfer between two systems -- as in the scientific language of the curriculum, yet may use a model of heat as entity-like in discussing particular phenomena with children.

How can we use the research information on naive theories to become better teachers and supporters of math and science learning?

This research alerts us to the need to be *aware* of students' naive theories and how they influence learning and alerts us to particular common misconceptions to look for. It provides some *tools* for becoming aware of and addressing children's misconceptions.

The wealth of research on misconceptions provides an important resource to teachers about common misconceptions on particular topics.[32] Misunderstandings have been identified as well as stages students progress through as they are helped to see the inadequacies of their current view. This work alerts teachers to common misconceptions that arise as they try to teach particular concepts -- making the task of addressing misconceptions a bit less daunting. It also helps teachers get a sense of the many different types of theories a child might construct and how these relate to the views held by scientists and mathematicians. Teachers and parents may identify some of their own misconceptions through this research!

It is important to stress that intuitive theories should not be discouraged. Instead, they should be viewed as the modeling clay from which the child and teacher together work to shape new understandings. We need to listen to how students are making sense of concepts. Engaging children in dialogue helps reveal idiosyncratic conceptions. Socratic discussion and questioning may help students to identify inconsistencies in their thinking through the questions of others.[33] Encouraging children to apply their ideas in a variety of different ways can help them to identify inconsistencies as well as discover new applications and connections.

How can Children be Taught Scientific Views of the World?

1. Lessons need to start with the ideas that children hold. The *current understandings* must be *revealed*.

2. These ideas need to be *explored* as possible solutions.

3. Children need to be *confronted* with ideas that are *discrepant* with their ideas. It is important to provide evidence that is contrary to their expectations on something that they care about.

4. Children need opportunities to *compare* the differences between the ideas.

5. Children need opportunities to *connect* the new idea broadly to the world. Otherwise they may see it as an isolated case.

Teachers who are alert to the nature of intuitive theories recognize the importance of becoming better listeners to children's "science and math talk." They attempt to reveal a child's understanding so that they know what types of interactions further the growth of the particular learner. They understand the necessity of asking questions to see how children construe terms rather than assuming that a certain term is intended as scientists would use it. They listen for the children's sense-making, consider how it relates to scientific and mathematic sense, and help students on a path to new understandings.

Children must see how scientific and mathematical theories make better sense than their naive constructions.[34] They must see that their present ideas are unsatisfactory in order to adopt new ones presented by the teacher.[35] Helping the child discover evidence that is discrepant with what a child's naive theory would predict, can help to introduce dissonance[36] in the child's mind. Immediate feedback as to the efficacy of a child's prediction helps to emphasize the discrepancy.[37] Helping the learner recognize that the information actually is discrepant can sometimes be a challenge -- particularly given children's tendency towards customized explanations as discussed earlier. Modeling abstract phenomena in concrete ways and helping children model it makes it more accessible. Beyond recognizing the discrepancy, the child must minimally understand the new theory, must see it as plausible, and must view it as suggesting the promise for greater explanatory power.[38]

Some researchers suggest[39] that computer simulated microworlds[40] are a promising way of addressing misconceptions -- providing discrepant results immediately. For instance, one program presents a set of computer simulations designed to help students learn the laws of motion providing a concrete means for a student to construct a model of an abstract concept. It encourages students to "ask" questions based upon their predictions because they are able to try out their theories. However, researchers warn students must see the connections between the microworlds and the real world. In some cases, students have revealed that they don't understand the abstract notion of a model well enough for the simulation to be useful.[41]

This body of research suggests changes in the format of textbooks and "teacher's editions" as well. Instead of communicating information in one direction, these resources need to encourage dialogue. Language and models in textbooks need careful review for possible misunderstandings that they can foster.[42] Increasingly, teacher editions and resource manuals contain information about misconceptions and how to address them. They may include examples of children's science or math talk and give a sense of questions teachers might ask in response[43] or how to interpret a child's understanding.

In summary, revealing children's intuitive theories and helping children build new understanding requires a different way of thinking about teaching and learning. The message here goes beyond focusing on the theories that children are creating in

their heads and encouraging children to talk about the sense that they are making. It points to a very basic need to help children see school learning as something that has to connect to their private learning. We need to take down the barriers by starting with what makes sense to them -- their evolving, personal understandings -- and building scientific and mathematical understandings from these.

For Further Information

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