NOTE: This essay is formatted so that it can be printed as a booklet for teachers to copy and share with colleagues. (Choose booklet printing so that the order of the pages is correct.)

Math/Science Matters: Resource Booklets on Research in Math and Science Learning

Booklet 2
Issues of Instructional Technique in Math and Science Learning

Teaching Thinking Skills: Does it Add Up for Math and Science Learning?

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Summary Points

Research on the effectiveness of teaching thinking skills has:

?shown promising results.
?led to more informed questions.

Current questions about teaching thinking are concerned with:

?teaching generic vs. situated thinking skills.
?the degree of explicitness of teaching skills.
?how to encourage use of the skills beyond the initial learning situation.

Which approaches appear most helpful in teaching mathematical and scientific thinking?

?explicitly attending to thinking processes.
?teaching of, for, and about thinking.
?centering learning around opportunities for thinking in the curriculum.
?attending to motivational as well as cognitive factors.
?valuing thinking in the cognitive economy of the classroom.

What thinking skills are relevant to science and math? Does the direct teaching of thinking skills benefit children's math and science learning?

"Think!" cries a science teacher to a student who stares blankly at lab data. "Think, I know you can do it!" a mathematics teacher says, trying to persuade a group of students faced with a complex word problem. The students respond by furrowing their brows and creating an appearance of deep concentration, but what is going on in their minds? Students do need to be able to think about what they learn. However, it takes more than exhortations or encouragement for students to be able to learn what good thinking in math and science looks like. We need to help them understand what it looks like and learn how to use good thinking. This essay explores what the research tells us about effective mathematical and scientific thinking and how to help students learn it.

What should an effective scientific or mathematical thinker be able to do?

Of the thinking skills recommended by educational experts, many are central to mathematical and scientific thinking ? for example, predicting, hypothesizing, information gathering, decision making, problem solving, comparing/contrasting, organizing, analyzing, inferring, and evaluating.[1] The National Council of Teachers of Mathematics calls for thinking skills such as conjecturing, thinking through novel situations, problem-solving, gathering evidence, building convincing arguments, and analyzing graphs.[2] The American Association for the Advancement of Science claims that students must learn to evaluate competing scientific claims in order to function well in our society.[3] Thus, there is considerable agreement that thinking skills are an integral part of science and mathematics.

Do we need to teach thinking skills directly for students to learn them?

Despite the importance of thinking skills to science and mathematics, it is valid to ask, "Do we really need to teach thinking skills? Won't students learn them anyway?" After all, some will argue, "Isn't it possible that students will learn to think on their own when presented with the proper sorts of problems?" The research evidence suggests that most students will *not* learn thinking skills *without explicit attention* to helping them do so.[4] In fact, people often display patterns of weak thinking.[5] For instance, people often fail to fully examine cases that are on the opposite side of an argument than one's own, and people often look for solutions to a problem before one has fully explored the nature of a problem.[6] People find certain types of reasoning to be difficult, such as making judgments involving elements of uncertainty or considering the influence of bias.[7] Clearly, if we want students to be good thinkers, we need to pay explicit attention to helping them learn what good thinking looks like and how to do it rather than just engaging children in thinking tasks.

Can thinking actually be taught?

Studies on the teaching of thinking show that, "elements of thinking are clearly teachable."[8] There has been some success in teaching deductive and inductive reasoning,[9] metacognitive strategies (thinking about thinking),[10] and problem solving.[11] For instance, a pair of researchers taught the patterns of thinking in science, such as the isolation and control of variables. Students focused on examining their assumptions, metacognition, and transferring of knowledge and strategies between contexts. Performance for many students on math, science, and English achievement tests significantly improved and persisted when measured again two years later.[12] Another researcher successfully taught heuristics for mathematical problem-solving. Students performed significantly better. When explicitly taught to self-monitor their thinking, students approached problems more systematically and thoughtfully. They were more likely to sort mathematical problems according to the deep structure (as experts do) than based on surface similarities (as novices do) than other students.[13] In another study, efforts to help students develop arguments that more fully explore the case opposite their own demonstrated marked improvement on that particular measure.[14]

While the teaching of thinking appears promising, reviewers of several studies caution that some findings can be difficult to interpret.[15] For instance, some research was conducted by the creator of the thinking skills program; some studies have inadequate control groups; and others fail to examine long-term benefits of training and the application of the training to other areas. The success of the programs depends on many implementation-specific variables, such as the quality of the teaching, administrative support, appropriateness of the program for the population taught, and the extent of implementation.[16]

What are the best ways to teach thinking?

While thinking can clearly be taught, questions remain about the best ways to do so: 1) Should educators teach thinking skills that are situation-specific, that is, applicable to a given context and well-suited to that context, but not broadly applicable? Or should they teach generic rules, that is, general rules that can be applied less exactingly to a variety of circumstances?; and 2) How can educators help students learn to use skills beyond the initial learning situation ? to "transfer" the skills? These questions are related and each is explored in turn below.

1. *Situation-Specific or General Rules?* The issue here is whether educators should teach rules as they emerge from and are shaped by a given context or whether they should teach general rules that could then be applied in a variety of circumstances. For instance, a science teacher could teach how to analyze fossil evidence as a situated-thinking skill. This means that she would teach students how to analyze fossil evidence by asking questions specific to the situation, such as where the fossils were found, what materials they were found in, and how they were arranged. Or that same

teacher could conduct a lesson on what is generally involved in analyzing evidence and could teach a series of steps for doing so. She would teach students to ask general questions, such as where the evidence comes from, what patterns and relationships can be construed from it, and what sources of error might exist in the information and in possible interpretations, and so forth.

This is a more difficult issue than it appears at first glance. While researchers have expressed cautious optimism about the teaching of generic thinking skills,[17] some reasoning appears to be fairly domain-specific ? the patterns of reasoning displayed by experts look quite different from those of novices.[18] There are many different levels of skills to teach, from very general to very situation-specific. Which level is most helpful to students depends a lot upon the circumstances.

Some general skills apply very straightforwardly and usefully to science and math situations; but some science and math situations require more specialized skills. Situation-specific skills are often specialized versions of more generalized skills ? so knowing the more general skills can be a step towards learning the specialized ones. In approaching a new problem ? one with which you have little situation-specific knowledge to draw upon ? generic approaches often provide an entry point for thinking about the problem.

Teaching only generalized skills presents problems of how to "transfer" or use the information in different sets of circumstances. Also, teaching generic skills often becomes an add-on in an already crowded curriculum, thus competing for valuable learning time. (One way to address this issue is to use generic thinking skills to think about curriculum content as a way of building understanding.[19] This "infusion" or "thinking-centered" approach is discussed further below.)

Given these issues, it seems sensible to strike a balance between situated and generic thinking skills so as to reap the benefits of each as much as possible. Whether generic or situated, explicit attention to skills ensures that students learn them.[20]

2. *What about transfer?* Another issue in the teaching of thinking has to do with the problem of transfer or how to get students to use what they know beyond the initial set of circumstances in which they learned it. Transferring thinking skills to new situations is critical if thinking skills are to have any real and lasting effect on learning. Critical as it is, transferring skills is not easy. Research shows that even when students are able to demonstrate mastery of certain skills, they are unlikely to transfer these skills to new areas of learning on their own.[21] However, students are able to apply the skills when their performance is "scaffolded" or explicitly supported.[22] Teachers who help students transfer information from thinking skills programs to other areas are more likely to see gains in those areas.[23] Students need help "bridging" or making outreaching connections to new

material.[24]

Educational researcher David Perkins makes a distinction between two types of transfer, "low-road" and "high-road transfer."[25] Teachers should attend to helping students with low- and high-road transfer depending upon the situation.

Low-road transfer has an automatic, reflexive quality. Routines that are well-practiced are automatically triggered in situations where there is a great deal of similarity between the two contexts. Examples of this type of transfer are using video game skills learned in one game in a new game or using reading skills in science.[26]

High-road transfer requires reflective thinking and direct attempts to make connections. The student learns something, abstracts the principles from it, and then applies it elsewhere (forward-reaching) or searches in memory for matches (backward-reaching). Deeper analogies are sought ? looking past surface similarities, for instance, noticing that different systems of measurement each have a feature called a standard of measure and that these serve a similar purpose despite surface differences in the systems. People are not particularly good at noticing analogies.[27] They need help finding them as well as seeing how some are better than others.[28]

Students are more likely to transfer thinking skills if they are motivated to use the skills they have learned, if they see the value of the skills in their own lives. Greater thoughtfulness should be rewarded with deeper, more useful understanding. Unless the rewards are clear, students won't be inclined to use the skills.

What should educators do about teaching thinking based on the available research?

Research on thinking has led to more informed and encompassing approaches to teaching thinking. For instance, there is increasing recognition of the need to teach *for*, *of*, and *about* thinking.[29] To teach *for* thinking means that the environment encourages thinking. The teaching *of* thinking involves the direct teaching of thinking skills. Teaching *about* thinking means helping children to understand information about our thinking processes, what is known about brain function, for example ? providing the rationale for particular approaches.

? Teaching *for* thinking. Teachers can create environments that encourage good thinking. The classroom practices that form the rituals, pace, and habits of a classroom have been shown to increase student achievement.[30] The "cognitive economy"[31] of the classroom has to value higher-order thinking. This means: time is made for thinking; deep understanding is valued over coverage of material; opportunities are taken to explore examples of thinking processes; and naturally occurring problems are capitalized upon to explore thought. For example, when the class needs to make a decision together, the steps for good decision-making are

used. Or when a disagreement erupts, the parties work together to evaluate evidence on each side of the argument. Or when a teacher poses a question, she pauses to give students time to think, which increases the number of thoughtful, higher-order responses[32] and rewards thoughtfulness rather than impulsivity.

Research shows that teachers' responses to students influence student behavior more than telling students what to do.[33] Teacher responses, or *enabling behaviors*, linked to student achievement include: asking higher-level cognitive questions,[34] modeling one's own thinking out loud so that students see their teacher engaged in thinking,[35] asking students to elaborate on comments,[36] or to clarify their ideas.[37] Asking students to clarify their ideas has been shown to increase metacognitive awareness ? students' ability to think about thinking.[38]

? Teaching *of* thinking. Thinking-centered learning or "infusion" approaches help address the issues of transfer and situated versus generic learning of skills. This approach uses thinking skills to teach curriculum content. Teachers and students learn to identify "thinkpoints" in the curriculum ? places where using the skills of good thinking will help them to think and learn the curriculum content more deeply. For example, one would look for "decision points," "evidence evaluation points," or "cause and effect points" ? places in the curriculum where a decision needs to be or has been made, evidence exists that needs to be or has been evaluated, or a tracing of patterns of cause would illuminate or has illuminated a certain set of events. Then a generic thinking skill is explicitly taught using the content of the curriculum.[39] The generic thinking tool helps students think in order to learn more deeply and the specific situation provides a context to teach a general skill that will be useful elsewhere. As students use the skills in different situations, they begin to learn the nuances of how particular applications transform the generic skill.

What would a thinking-centered lesson look like? Let's explore an "evidence evaluation point" as an example. At the point in the curriculum when the story of Copernicus is taught, a class could delve in deeply using the skills of evidence-evaluation to consider what evidence existed at the time and the reasons and beliefs that gave rise to the events that transpired. Students would evaluate evidence on both sides of the case, consider sources of possible bias that influenced people at the time, and so forth. Teachers would explicitly teach steps and/or criteria for doing a good job evaluating evidence. Using the thinking skill well, in this case, evaluating evidence fairly, often pushes students to seek out more information and paves the way to deeper understanding. Rather than a random gathering of facts, students are informing a certain set of questions ? leading to a thoughtful, more connected search. Teachers then attend to helping students transfer the thinking skill to other situations. Students could consider what current scenarios exist in which a scientist

takes a bold, perhaps unpopular, stance and the population reacts in such a way that evidence on both sides of the case should be carefully evaluated.

Thinking-centered learning also provides opportunities to examine misconceptions about thinking and pitfalls of poor thinking. For instance, good scientific thinking entails an openness and appreciation for new ideas balanced with a skepticism and appreciation for logic and evidence. However, people often fail to be as open-minded about evidence on the side of a case opposite their own. Students would consider how this pitfall contributed to the events in the Copernicus story.

? Teaching *about* thinking. Teaching about thinking helps to deal with issues of relevance and motivation. If students understand why a certain technique works in light of what we know about the memory process and the anatomy of our brains, it supports their tendency to use the technique. It encourages metacognition or a greater self-reflectiveness about one's thinking processes. This should result in deeper self-understanding.

What does teaching about thinking look like? For instance, if a teacher wanted students to understand the importance of making connections in helping them to remember what they've learned, she might do the following. She could first give out a list of science words or math equations that have little relationship to each other and give students a certain amount of time to memorize them. After seeing how many they can recall, she would then give a second list, one where students will find it easier to draw connections between the concepts, and give students the same amount of time to memorize them. After seeing how many they can recall, students could compare their performance on each list and the strategies they used in each instance.

In summary, the research on the teaching of thinking has helped to raise some important issues. There are important thinking skills embedded in science and mathematics that need to be taught explicitly or students will not learn them. Teachers should use a combination of generic and situated skills so that students have a set of skills to approach novel problems but also understand the nuances involved in thinking within particular science and math situations. Teachers should encourage both low-road and high-road transfer in the appropriate contexts. Finally, teachers should establish a classroom environment where thinking is valued and the curriculum offers many opportunities to use the skills of good thinking to learn deeply. While there is certainly more to be learned about the teaching of good thinking, we know enough right now to do a lot to teach students to become good mathematical and scientific thinkers.

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